Mark Scheme (Results)

June 2008
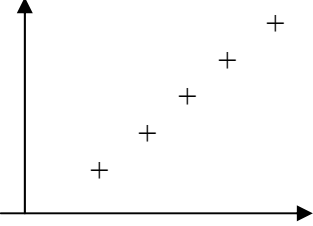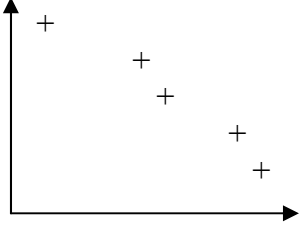
GCE Mathematics (669101)

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Question number	Scheme		Marks
1. (a)	$\bar{x} = \left(\dfrac{6046}{36} = \right)167.94...$ awrt **168**		B1
	$s^2 = \dfrac{1016338 - 36 \times \bar{x}^2}{35}$		M1
	$= 27.0253....$ awrt **27.0** (Accept 27)		A1 (3)
(b)	99% Confidence Interval is: $\bar{x} \pm 2.5758 \times \dfrac{5.1}{\sqrt{36}}$		M1A1ft
	2.5758		B1
	$= (165.755..., 170.133...)$ awrt **(166,170)**		A1 A1 (5)
			8 marks
(a)	M1 for a correct expression for s^2, follow through their mean, beware it is very "sensitive"		
	$\begin{aligned}167.94 &\to \dfrac{999.63..}{35} \to 28.56...\\ 167.9\ &\to \dfrac{1483.24..}{35} \to 42.37...\\ 168\ \ &\to \dfrac{274}{35}\ \ \ \to 7.82\end{aligned}$ $\boxed{\begin{array}{l}\text{These would all}\\ \text{score M1A0}\end{array}}$		
	Use of 36 as the divisor (= 26.3…) is M0A0		
(b)	M1 for substituting their values in $\bar{x} \pm z \times \dfrac{5.1 \text{ or } s}{\sqrt{36}}$ where z is a recognizable value from tables		
	1st A1 follow through their mean and their z (to 2dp) in $\bar{x} \pm z \times \dfrac{5.1}{\sqrt{36}}$		
	Beware: $167.94 \pm 2.5758 \times \dfrac{5.1^2}{36} \to (166.07..,169.8..)$ but scores B1M0A0A0A0		
	Correct answer only in (b) scores 0/5		
	2nd & 3rd A marks depend upon 2.5758 **and** M mark.		

Question number	Scheme	Marks
2.	$\dfrac{115 \times 70}{217} = 37.0967...$ or $\dfrac{1150}{31}$ (etc) $\dfrac{1265}{31}, \dfrac{1020}{31}, \dfrac{1122}{31}$	M1

Expected (Obs)	A	S	H
Boy	37.1 (30)	37.1 (50)	40.8 (35)
Girl	32.9 (40)	32.9 (20)	36.2 (42)

A1A1

H_0 : There is no association between course and gender

H_1 : There is some association between course and gender   (both)   B1

$$\sum \frac{(O-E)^2}{E} = \frac{(37.1-30)^2}{37.1} + \frac{(32.9-40)^2}{32.9} + ... + \frac{(36.2-42)^2}{36.2}$$ M1A1ft

$= 1.358 + 4.485 + 0.824 + 1.532 + 5.058 + 0.929 = 14.189...$ awrt **14.2**   A1

$\nu = (3-1)(2-1) = 2$, χ_2^2(1%) critical value is 9.210   (condone 9.21)   B1, B1ft

Significant result   or   reject null hypothesis   M1

There is evidence of an association between course taken and gender   A1ft   (11)

          [Correct answers only score full marks]   **11 marks**

ALT	$\sum \dfrac{O^2}{E} - N = \dfrac{30^2}{37.1} + \dfrac{40^2}{32.9} + ... + \dfrac{42^2}{36.2} - 217$	M1A1ft

1st M1 for some use of the $\dfrac{\text{row total} \times \text{col total}}{\text{grand total}}$ formula

1st A1 for one correct row or one correct column of expected frequencies to nearest integer

2nd A1 for all expected frequencies correct to awrt 1 dp (Allow exact fractions)

1st B1 for hypotheses. Independence is OK. Must mention courses and gender at least once.

          Use of ρ or "correlation" is B0 but allow ISW.

2nd M1 for an attempt to calculate test statistic. At least one correct expression, ft expected freq.

3rd A1 follow through expected frequencies for at least 3 expressions

3rd M1 for a correct statement relating their test statistic and their cv (may be implied by comment)

5th A1 for a contextualised comment relating their test statistic and their cv. Ignore their H_0 or H_1

          or assume that they were correct.   Must mention courses and gender

Question number	Scheme	Marks
3. (a)	(i) (ii)	(i) B1 (ii) B1B1 (3)

(b)(i)

	A	*B*	*C*	*D*	*E*	*F*	*G*
Rank (Judge 1)	1	4	2	3	5	6	7
Rank (Judge 2)	1	2	4	3	5	7	6
d^2	0	4	4	0	0	1	1

$\sum d^2 = 10$ M1A1

$$r_s = 1 - \frac{6\times 10}{7\times(49-1)} = 1 - \frac{5}{28} = \frac{23}{28} \qquad \text{or} \qquad \text{awrt } \mathbf{0.821}$$ M1A1 (6)

(ii) $H_0 : \rho = 0 \qquad H_1 : \rho > 0 \qquad$ (Allow ρ_S) $\qquad$ ($H_1 : \rho \neq 0$ scores B0)   B1,B1

r_s 5% one tail critical value is **0.7143**   B1

Significant result $\qquad$ or $\qquad$ reject null hypothesis   M1

There is evidence of a (positive) correlation between the judges <u>or</u> the judges agree   A1ft (5)

14 marks

(a) (i)	1st B1 for 5 or more points on a straight line of positive gradient

(ii) 2nd B1 for 4 or more points satisfying $-1 < r < 0$

3rd B1 for 5 or more points of decreasing ranks not on a straight line

(b)(i) 1st M1 for attempting to rank one of the judges (at least 2 correct rankings)

2nd M1 for ranking both (may be reversed) (at least 2 correct rankings)

3rd M1 for attempting d^2.

1st A1 for $\sum d^2 = 10$

4th M1 for correct use of the r_s formula

(ii) 3rd B1 for the correct critical value - depends upon their H_1 : $\rho > 0$ needs 0.7143, $\rho \neq 0$, 0.7857

The H_1 may be in words so B0B1 is possible. If no H_1 award for 0.7143 only.

5th M1 for a correct statement relating their r_s and their cv (may be implied by correct comment)

3rd A1ft $\qquad$ follow through their r_s and their cv. Comment in context. Must mention judges.

Don't insist on "positive" and condone it if they are using $\rho \neq 0$.

Question number	Scheme	Marks
4. (a)	$X = M_1 + M_2 + M_3 + M_4 \sim N(336, 22^2)$ $\hfill \mu = \mathbf{336}$	B1
	$\hfill \sigma^2 = 22^2 \text{ or } \mathbf{484}$	B1
	$P(X < 350) = \quad P\left(Z < \dfrac{350 - 336}{22}\right)$	M1
	$= P(Z < 0.64) \hfill \text{awrt } \mathbf{0.64}$	A1
	$= \hfill \text{awrt } \mathbf{0.738} \text{ or } \mathbf{0.739}$	A1 $\qquad$ (5)
(b)	$M \sim N(84, 121)$ and $W \sim N(62, 100)$ $\quad$ Let $Y = M - 1.5W$	M1
	$E(Y) = 84 - 1.5 \times 62 = -9$	A1
	$Var(Y) = Var(M) + 1.5^2 Var(W)$	M1
	$= 11^2 + 1.5^2 \times 10^2 = 346$	A1
	$P(Y < 0)\,, \qquad = P(Z < 0.48\ldots) = \hfill \text{awrt } \mathbf{0.684} \sim \mathbf{0.686}$	M1, A1 $\quad$ (6)
		11 marks
(a)	2nd B1 for $\sigma = 22$ or $\sigma^2 = 22^2$ or 484 M1 $\quad$ for standardising with their mean and standard deviation (ignore direction of inequality)	
(b)	1st M1 for attempting to find Y. Need to see $\pm(M - 1.5W)$ or equiv. May be implied by $Var(Y)$. 1st A1 for a correct value for their $E(Y)$ i.e. usually ± 9. Do not give M1A1 for a "lucky" ± 9. 2nd M1 for attempting $Var(Y)$ e.g. $\ldots + 1.5^2 \times 10^2$ or $11^2 + 1.5^2 \times \ldots$ 3rd M1 for attempt to calculate the correct probability. Must be attempting a probability > 0.5. $\qquad$ Must attempt to standardise with a relevant mean and standard deviation $\qquad$ Using $\sigma^2_M = 11$ or $\sigma^2_W = 10$ is not a misread.	

Question number	Scheme	Marks
5. (a)	Only cleaners - no managers i.e. not all <u>types</u>. OR Not a random sample 1st 50 may be in same shift/group/share <u>same views</u>. OR Not a random sample (Allow "not a representative sample" in place of "not a random sample")	B1g B1h (2)
(b)(i)	Label employees (1-550) or obtain an ordered list Select <u>first</u> using <u>random numbers</u> (from 1 - 11) Then select every 11th person from the list	B1 B1 B1
(ii)	Label managers (1-55) and cleaners (1-495) Use random numbers to select… …5 managers and 45 cleaners	M1 M1 A1 (6)
(c)	390, 372 (They must be in this order)	B1, B1 (2) **10 marks**

After 1st B1, comments should be in **context**, i.e. mention cleaners, managers, types of worker etc

(a)   1st B1g   for one row

2nd B1h   for both rows. "Not a random sample" only counts once.

Score B1B0 or B1B1 or B0B0 on EPEN

(b)(i)   1st B1   for idea of labelling or getting an ordered list. No need to see 1-550.

2nd B1   selecting first member of sample using random numbers (1-11 need not be mentioned)

3rd B1   selecting every nth where n = 11.

(ii)   1st M1   for idea of <u>two</u> groups and labelling <u>both</u> groups. (Actual numbers used not required)

2nd M1   for use of random numbers within each strata. Don't give for SRS from all 550.

"Assign random numbers to managers and cleaners" scores M0M1

A1   for 5 managers <u>and</u> 45 cleaners. (This mark is dependent upon scoring at least one M)

6

Question number	Scheme	Marks
6. (a)	$p = \dfrac{0 \times 11 + 1 \times 21 + ...}{10 \times (11 + 21 + ...) \text{ or } 10 \times 100}, = \dfrac{223}{1000} = 0.223$ **(*)** (Accept $\dfrac{223}{1000}$)	M1, A1cso (2)
(b)	$r = (0.8)^{10} \times 100 = 10.7374$ awrt **10.74**	M1A1
	$s = \dbinom{10}{2}(0.8)^8 \times (0.2)^2 \times 100 = 30.198...$ awrt **30.2**	A1
	$t = 100 - [r + s + 26.84 + 20.13 + 8.81] =$ awrt **3.28**	A1cao (4)
(c)	H_0 : Binomial ($[n = 10]$, $p = 0.2$) is a suitable model for these data	B1
	H_1 : Binomial ($[n = 10]$, $p = 0.2$) is NOT a suitable model for these data	B1 (2)
(d)	Since $t < 5$, the last two groups are combined	M1
	and $v = 4 = 5 - 1$	A1 (2)
(e)	Critical value $\chi_4^2(5\%) = 9.488$	B1
	Not significant or do not reject null hypothesis	M1
	The binomial distribution with $p = 0.2$ is a suitable model for the number of	
	cuttings that do not grow	A1 (3)
		13 marks
(a)	M1 Must show clearly how to get either 223 or 1000. As printed or better.	
	A1cso for showing how to get <u>both</u> 223 and 1000 and reaching $p = 0.223$	
(b)	M1 for any correct method (a correct expression) seen for r or s.	
	1st A1 for correct value for r awrt 10.74	
	2nd A1 for s = awrt 30.2	
	3rd A1 for t = 3.28 only	
(c)	B1 for each. The value of p must be mentioned at least once. Accept B(10, 0.2)	
	If hypotheses are correct but with no value of p then score B0B1	
	Minimum is $X{\sim}$B(10, 0.2). If just B(10, 0.2) and not B(10, 0.2) award B1B0	
(d)	M1 for combining groups (must be stated or implied by a new table with combined cell seen)	
	A1 for the calculation 4 = 5 - 1	
(e)	M1 for a correct statement based on 4.17 and their cv(context not required) (may be implied)	
	Use of 4.17 as a critical value scores B0M0A0	
	A1 for a correct interpretation in context and p =0.2 and cuttings mentioned.	

Question number	Scheme	Marks
7. (a)	$H_0 : \mu_F = \mu_M \qquad H_1 : \mu_F \neq \mu_M \quad$ (Allow μ_1 and μ_2)	B1
	$z = \dfrac{6.86 - 5.48}{\sqrt{\dfrac{4.51^2}{200} + \dfrac{3.62^2}{100}}}$	M1 A1
	$= 2.860\ldots$ $\qquad\qquad\qquad\qquad\qquad\qquad$ awrt ($\pm$)**2.86**	A1
	2 tail 5% critical value ($\pm$) 1.96 $\qquad$ (or probability awrt 0.0021~0.0022)	B1
	Significant result $\quad$ or $\quad$ reject the null hypothesis $\quad$ (o.e.)	M1
	There is evidence of a difference in the (mean) amount spent on junk food by male and female teenagers	A1ft $\qquad$ (7)
(b)	CLT enables us to assume $\bar{F}$ and $\bar{M}$ are normally distributed	B1 $\qquad$ (1)
		8 marks
(a)	1st M1 for an attempt at $\dfrac{a-b}{\sqrt{\dfrac{c}{100 \text{ or } 200} + \dfrac{d}{100 \text{ or } 200}}}$ with 3 of a, b, c or d correct	
	1st A1 for a fully correct expression	
	2nd B1 for $\pm$ 1.96 <u>but</u> only if their H_1 is two-tail (it may be in words so B0B1 is OK)	
	$\qquad\quad$ If H_1 is one-tail this is automatically B0 too.	
	2nd M1 for a correct statement based on comparison of their z with their cv. May be implied	
	3rd A1 for a correct conclusion in context based on their z and 1.96.	
	$\qquad\quad$ Must mention <u>junk food</u> or <u>money</u> and <u>male vs female</u>.	
(b)	B1 $\qquad$ for $\bar{F}$ or $\bar{M}$ mentioned. Allow "<u>mean</u> (amount spent on junk food) is <u>normally distributed</u>"	
	$\qquad\qquad$ Read the whole statement e.g. " original distribution is normal so mean is…" scores B0	